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To the Problem of a Passive Levitation of Bodies in Physical Fields

A possibility of levitation of a body carrying a point electrical charge in the field of a fixed point charge of the same sign is shown. Stabilization of an unstable equilibrium, when gravity is compensated by Coulomb's force, is realized using gyroscopic forces generated due to the rotation of the body. A finite range of angular velocity corresponds to conservative stability of the levitating body. It is shown that dissipative and circulation forces introduced into consideration simultaneously can improve the stability of the system to asymptotic. Dependence of the domain of attraction of stable equilibrium on parameters of the system is studied numerically. [DOI: 10.1115/1.4000384]

The equilibrium of an electric charge, when gravity acting on it is compensated by the Coulomb force from another fixed point charge of the same sign, is unstable. This follows from Earnshaw's theorem [1], which states that it is impossible to find a static configuration of charged particles governed by the inverse square law forces. Kelvin's theorem about gyroscopic stabilization of a conservative system with an even degree of instability shows one of the possible ways to achieve levitation of an electric charge in an electrostatic field. Sufficiently, strong magnetic field, with magnetic field vector oriented vertically, creates gyroscopic force and brings the system of charges to conservative stability [2]. One could think that, upon rigidly connecting the charge to be suspended to a rigid body, levitation of the body with a point charge could be realized by the means of sufficiently quick rotation. However, in this case, the matrix of gyroscopic forces becomes degenerate: The number of degrees of freedom increases, and gyroscopic forces correspond only to the angular degrees of freedom. Therefore, Kelvin's theorem, proof of which is based on the assumption that the determinant of the matrix of gyroscopic forces is not equal to zero [3], does not answer the question about the possibility of gyroscopic stabilization of the system. In this paper, the conditions of stable "hovering" of a point charge rigidly connected to an axisymmetric rotating rigid body (Fig. 1) in electrostatic field are found.

The problem discussed here and the problem of stabilization of a permanent magnet suspended in the field of another permanent magnet (magnetic Levitron [4]), considered, in particular, in the works in Refs. [5–7], have a common nature: The translational motions of the suspended body are coupled with its angular motion, and the rotation of the body creates gyroscopic forces only in the equations of angular motion. Among other possible models of Levitron, the one presented here, accentuates all possible instabilities most effectively. This work also differs by the more simple derivation of stability conditions and by indication of small nonconservative forces improving the stability to the asymptotic.

1 Problem Statement: Equations of Motion

A symmetric body carries a point electrical charge q located on the axis of symmetry at distance $|a|$ from the center of mass (Fig. 1). The body is suspended in the field of another fixed point charge of the same sign, and the body is brought in the state of rotation around the axis of symmetry with the initial angular velocity Ω_0. Let us denote the radius vector of center of inertia by $\mathbf{r}$ and the position of the charge connected to the body by $\mathbf{r_1}$. The relationship between these two vectors is determined by equality

$$\mathbf{r_1} = \mathbf{r} + \mathbf{a}, \quad \text{where} \quad \mathbf{a} = -\mathbf{a}(\sin\alpha\cos\beta\mathbf{i} + \sin\beta\mathbf{j} + \cos\alpha\cos\beta\mathbf{k})$$

The positive value of $\mathbf{a}$ corresponds here to the case when, at the equilibrium of the system, the center of mass is located above the suspended charge (Fig. 1), while the negative $\mathbf{a}$ corresponds to the position of the center of mass below the charge. The angles α and β, characterizing the position of the vector $\mathbf{a}$ in the coordinate system xyz, are shown on Fig. 2 (axis z is directed vertically).

The Lagrange function of the system under consideration has the form

$$L = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)\frac{m}{2} + (\dot{\alpha}^2\cos^2\beta + \dot{\beta}^2)\frac{A}{2} + (\Omega + \dot{\beta}\sin\beta)^2\frac{C}{2} - \frac{q^2}{4\pi\varepsilon_0 r_1} - mgz \tag{1}$$

where A, C, and m are the moments of inertia and mass of the body as

$$r_1 = \sqrt{(x - a\sin\alpha\cos\beta)^2 + (y - a\sin\beta)^2 + (z - a\cos\alpha\cos\beta)^2}$$

The system of equations of the perturbed motion of the body, corresponding to this Lagrange function, after switching to the dimensionless variables and parameters, will have the form

$$\ddot{x} - (x - \delta\sin\alpha\cdot\cos\beta)\cdot f(x,y,z,z_0) = 0$$

$$\ddot{y} - (y - \delta\sin\beta)\cdot f(x,y,z,z_0) = 0$$

$$\ddot{z} + (z_0 - \delta)^{-2} - (z + z_0 - \delta\cos\alpha\cdot\cos\beta)\cdot f(x,y,z,z_0) = 0$$

$$\ddot{\alpha}\cos^2\beta + (I-2)\dot{\alpha}\dot{\beta}\cos\beta\cdot\sin\beta + I\dot{\varphi}\dot{\beta}\cos\beta - \delta\cos\beta\cdot(z\sin\alpha - x\cos\alpha)\cdot f(x,y,z,z_0) = 0$$

$$\ddot{\beta} + (1-I)\dot{\alpha}^2\sin\beta\cdot\cos\beta - I\dot{\varphi}\dot{\alpha}\cos\beta - \delta(x\sin\alpha\cdot\sin\beta - y\cos\beta - z\cos\alpha\cdot\sin\beta)\cdot f(x,y,z,z_0) = 0$$

$$\dot{\varphi} + \dot{\alpha}\sin\beta = \text{const} \tag{2}$$

Here, x, y, and z are the dimensionless deviations of the center of mass from the equilibrium $x_0=0$

$$y_0 = 0$$

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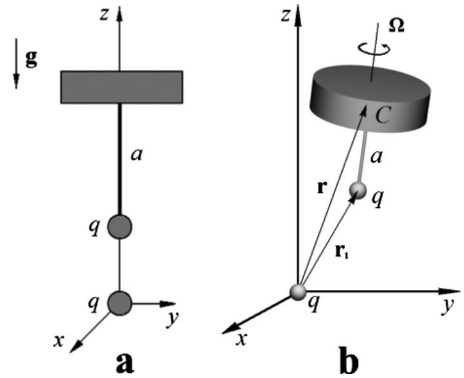

Fig. 1 A symmetric rigid body with an electric charge q rotates with angular speed $\dot{\varphi}$ in a field of an identical fixed electrical charge q located in the origin of the coordinate system. Radius vectors r and r_1 define the position of the center of mass of the levitating body and position of the charge connected with it correspondingly. (*a*) Equilibrium state of the system and (*b*) nonequilibrium state of the system.

$$z_0 = \delta + \frac{q}{l_* \sqrt{4\pi\varepsilon_0 mg}}$$

$$\alpha_0 = 0$$

$$\beta_0 = 0$$

$$\dot{\varphi}_0 = \Omega_0$$

Quantities $t_* = \sqrt[4]{q^2/4\pi\varepsilon_0 mg^3}$ and $l_* = \sqrt{A/m}$ were used as time and length scales during the transition to dimensionless variables. There was the introduced notation $\delta = a/l_*$ for the distance between the charge and the center of mass, and $I = C/A$ for the ratio of the moments of inertia.

The function $f(x,y,z,z_0)$ in Eq. (2) has the form

$$f(x,y,z,z_0) = [(x - \delta \sin \alpha \cdot \cos \beta)^2 + (y - \delta \sin \beta)^2 + (z_0 + z - \delta \cos \alpha \cdot \cos \beta)^2]^{-3/2}$$

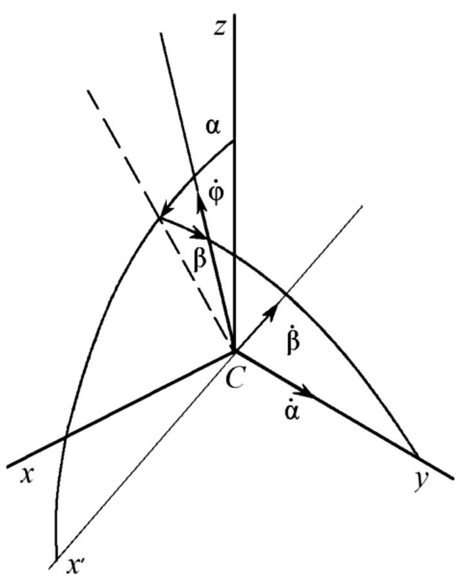

Fig. 2 A system of angular coordinates of the symmetry axis of levitating body

To study the motion under the condition of small deviations from the equilibrium linearizes the system of Eq. (2) as

$$\ddot{\alpha} = -\delta x - H\dot{\beta} + \chi\alpha$$

$$\ddot{\beta} = -\delta y + H\dot{\alpha} + \chi\beta$$

$$\ddot{x} = x - \delta\alpha$$

$$\ddot{y} = y - \delta\beta$$

$$\ddot{z} = -2z \quad (3)$$

where $H = C/A\Omega_0 = I\Omega_0$ and $\chi = z_0\delta$.

The equation for z separates. Thus, we can conclude that, in the vertical direction, the center of mass performs harmonic oscillations with the frequency, in the terms of initial variables equal to $\omega^2 = 2g\sqrt{4\pi\varepsilon_0 mg/q^2}$.

Without rotation, the system is unstable in each of the four degrees of freedom. At first, sight stabilization of a body due to its own rotation is impossible. When $\Omega_0 \neq 0$, gyroscopic forces are presented only in the equations of angular motion, and Kelvin's theorem about gyroscopic stabilization of a conservative system with an even degree of instability is inapplicable here since its proving implies that the gyroscopic forces matrix is not degenerate. However, the detailed analysis gives other results. A suspended body can be stabilized by rotation, but unlike the classical case of gyroscopic stabilization (Lagrange top), an interval of the angular speed, providing conservative stability, is limited both from below and from above.

2 Gyroscopic Stabilization of the System

Consider the following system of equations equivalent to the first four equations of system (2):

$$\ddot{u} - u + \delta v = 0$$

$$\ddot{v} - iH\dot{v} - \chi v + \delta u = 0 \quad (4)$$

where $u = x + iy$, $v = \alpha + i\beta$, and $i = \sqrt{-1}$.

Searching the solution in the form $u = A\exp(ipt)$ and $v = B\exp(ipt)$, the characteristic equation is obtained

$$p^4 - Hp^3 + (1+\chi)p^2 - Hp - \delta^2 + \chi = 0 \quad (5)$$

Let us establish if it is possible to indicate such values of H, δ, and χ, at which the four roots of this equation are real, i.e., system (3) becomes conservatively stable.

Let $\chi - \delta^2 > 0$. Under this condition, the equilibrium corresponds to the position of the center of mass of the top above the charge. Parameter H in Eq. (5) will be expressed as

$$H = \frac{p^4 + (1+\chi)p^2 + \chi - \delta^2}{p(p^2+1)} \quad (6)$$

The dependence $H(p)$ under the condition pointed above can be plotted as shown on Fig. 3(*a*). With the values of H belonging to the interval $[H_{min}, H_{max}]$ the characteristic Eq. (5) has four real roots. This is possible if the curve $H(p)$ has three extremums, i.e., the equation

$$p^6 + (2-\chi)p^4 + (1 - 2\chi + 3\delta^2)p^2 - (\chi - \delta^2) = 0 \quad (7)$$

Following from the condition, $dH/dp = 0$ has three real positive roots p^2. For that to be true, three sign inversions are required in the sequence of coefficients at degrees of $y = p^2$, i.e., alongside with $\chi - \delta^2 > 0$, the inequalities $2 - \chi < 0$, and $1 - 2\chi + 3\delta^2 > 0$ should hold.

It follows that at least $\chi > 2$ and $\delta^2 > 1$.

Particularly, under condition $\chi - \delta^2 \ll 1$, one of the roots of Eq. (7) is approximately $p_1^2 \approx \chi - \delta^2/1 + \chi$, while the rest two roots are $p_{2,3}^2 \approx \chi - 2 \pm \sqrt{\chi^2 - 8\chi}/2$. Therefore, $\chi > 8$.

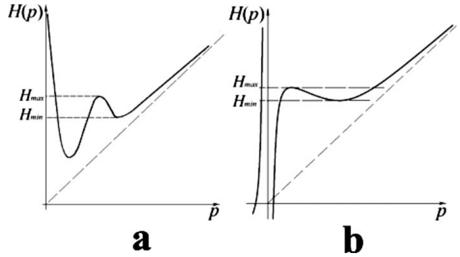

Fig. 3 Dependence of parameter *H*, characterizing the gyroscopic properties of a body on oscillation frequency and *p*: (*a*) under condition of $\chi-\delta^2>0$ and (*b*) under condition of $\chi-\delta^2<0$. The interval $[H_{\min},H_{\max}]$ is the region of stability.

Let us specify the possible values of χ and δ. The conditions that the roots of the cubic equation for p^2 are real. Equation (7) [8] is transformed to give

$$\delta^4-\frac{\chi^2+14\chi+1}{12}\delta^2+\frac{2}{27}(\chi+1)^3<0 \qquad (8)$$

This inequality has solutions if $(\chi^2+14\chi+1)^2/144>8/27(\chi+1)^3$. Hence, $\chi>5.3$. At this boundary point, inequality (8) holds only at $\delta^2=4.304$. For each value of $\chi>5.3$, δ^2 belongs to some interval, which can be determined from Eq. (8), e.g., at $\chi=10$, $8.546<\delta^2<11.538$.

Notice that the equilibrium is only possible with a sufficiently large negative rigidity, χ in equations for angular motions, and with a sufficiently large coefficient δ^2 coupling the equations of translational and angular motions.

Next, a particular case with $\chi=10$ and $\delta=3$ is considered. The equation for the points of extremum $H(p)$ is $p^6-8p^4+8p^2-1=0$. Its roots are $p_1^2=0.1459$, $p_2^2=1$, and $p_3^2=6.854$. The extreme values are $H_{\min}(p_1)\approx 6$, $H_{\max}(p_2)\approx 6.5$, and $H_{\min}(p_3)\approx 6$. Therefore, for all the roots of Eq. (3) to be real, the values of the parameter H should belong to the interval $6<H<6.5$, e.g., with $H=6.25$. Equation (5) has the form $p^4-6.25p^3+11p^2-6.25p+1=0$. Its solutions are $p_1\approx 0.268$, $p_2\approx 0.6054$, $p_3\approx 1.6466$, and $p_4\approx 3.73$.

The problem of determining conditions under which the quartic Eq. (5) with parameters χ, δ, and H has purely real roots and has reduced to a simpler one—determining conditions for real roots of the cubic Eq. (7) with parameters χ and δ. This is the way to determine the range of values of χ, δ, and H, within which the roots of Eq. (5) are real, i.e., the object under consideration is conservatively stable. On Fig. 4(*a*), sections of the stability region for the case $\chi-\delta^2>0$ by planes $H=$const with step 1 on segment $5\le H\le 10$ are marked by gray.

The analysis performed generalizes Kelvin's theorem on gyroscopic stabilization for the case when the determinant of gyroscopic forces matrix vanishes. Unlike the typical case, here, the variation range of the force parameter H, corresponding to the gyroscopic stabilization of the system, becomes bounded, both above and below.

In equilibrium under the condition $\delta<0$, the center of mass of the body is below the charge. Practically, it can be implemented, for example, in the form of a hollow cone with an electric charge at the vertex and a fixed charge inside. Dependence of H on p can have a form, as shown on Fig. 3(*b*). With H from the interval $[H_{\min},H_{\max}]$, Eq. (5) also has four real roots, one of which, in this case, is negative. The region of stability on plane δ,χ with $5\le H\le 10$ for $\chi-\delta^2<0$ is shown on Fig. 4(*b*).

3 Asymptotic Stabilization of the System by Dissipative and Circulation Forces

Though from the appearance of the object under consideration, it is hard to determine the possibility of its gyroscopic stabilization. It turned out that its conservative stability is attainable. Is it possible to achieve asymptotic stability of the system by introducing small nonconservative forces?

Consider the following transformed system of equations:

$$\ddot{u}-u+\delta v+h_1\dot{u}+i\kappa_1 u=0$$

$$\ddot{v}-iH\dot{v}-\chi v+\delta u+h_2\dot{v}+i\kappa_2 v=0 \qquad (9)$$

i.e., small dissipative and circulation forces with the coefficients $h_{1,2}\ll 1$ and $\kappa_{1,2}\ll 1$, respectively, are added into the initial Eq. (3).

Characteristic equation of the system has the form

$$F(p)=(p^2-1)(p^2-iHp-\chi)-\delta^2+(p^2-1)(h_2p+i\kappa_2)+(p^2-iHp-\chi)(h_1p+i\kappa_1)=0$$

Here, due to the smallness of $h_{1,2},\kappa_{1,2}$, quadratic terms are omitted.

To study stability using the D-partitioning method [9], assume $p=i\omega$ in $F(p)$ and set it to be equal to zero. The real and the imaginary parts of $F(i\omega)$ are

$$(\omega^2+1)(\omega^2-\omega H+\chi)-\delta^2=0$$

$$(\omega^2-qH+\chi)(h_1\omega+\kappa_1)+(\omega^2+1)(h_2\omega+\kappa_2)=0 \qquad (10)$$

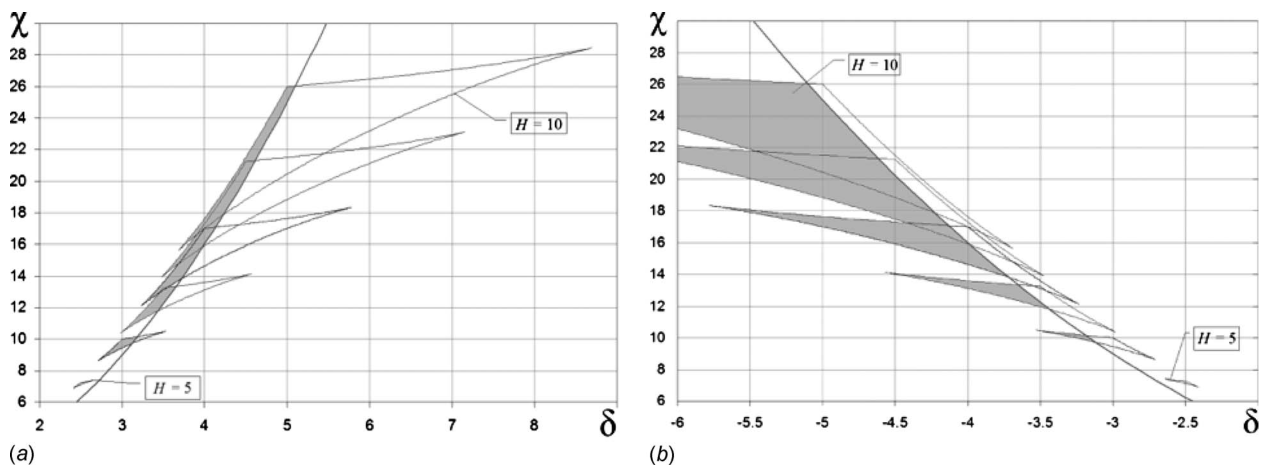

Fig. 4 The regions of stability of the parameter plane for different *H* from interval $5\le H\le 10$ with step 1: (*a*) with condition of $\chi-\delta^2>0$ and (*b*) with $\chi-\delta^2<0$

Choose k_1 and k_2 as the parameters on the plane of which we shall perform the D-partitioning. Usually, D-partitioning parameters are presented in both real and imaginary parts of $F(i\omega)$. With the linear inclusion of these parameters, two lines, whose intersection point with different values of ω from the interval ($-\infty,\infty$) provides the boundary of D-partitioning, correspond to the equations Re $F(i\omega)=0$ and Im $F(i\omega)=0$ on their planes. In this case, the parameters k_1 and k_2 enter linearly and only into the second equation. Therefore, a straight line on the plane of k_1 and k_2, called singular line, corresponds to each of $\omega=\omega_k(k=1,2,3,4)$, vanishing the real part of $F(i\omega)$. The imaginary part of the characteristic equation is transformed to the form

$$h_1\omega_k + \kappa_1 + \frac{(\omega_k^2+1)^2}{\delta^2}(h_2\omega_k + \kappa_2) = 0$$

$$(\omega_1 = 0.268, \quad \omega_2 = 0.6054, \quad \omega_3 = 1.6466, \quad \omega_4 = 3.73) \tag{11}$$

The set of singular lines partitions the plane of k_1 and k_2 into regions D_n with different numbers of roots with positive real parts (D_0 is the region of stability).

Assuming for simplicity $h_1=h_2=h$, we shall have the following singular lines:

$$\omega_1 \text{ corresponds to the line } \kappa_1 + 0.1276\kappa_2 + 0.3022h = 0$$

$$\omega_2: \quad \kappa_1 + 0.2075\kappa_2 + 0.731h = 0$$

$$\omega_3: \quad \kappa_1 + 1.5304\kappa_2 + 4.1666h = 0$$

$$\omega_4: \quad \kappa_1 + 24.711\kappa_2 + 95.9h = 0$$

One of the roots of the characteristic equation on each of these lines is purely imaginary and equals to $p_k=i\omega_k$.

With the small deviation of the parameters k_1 and k_2 from their values on the singular lines, $p_k=iq_k+\Delta p_k$ and $\Delta p_k \leqslant 1$. By substituting p_k into the characteristic equation, the following is obtained after simple transformations:

$$F'(\omega_k)\Delta p_k = (-4\omega_k^3 + 18.75\omega_k^2 - 22\omega_k + 6.25)\Delta p_k$$
$$= \frac{\delta^2}{(\omega_k^2+1)}\Delta\kappa_1 + (\omega_k^2+1)\Delta\kappa_2 \tag{12}$$

It follows that the small increment Δp_k is real, and its sign depends on $F'(\omega_k)$ and also on which side from the singular line the deflection occurred. Thus, for the singular line corresponding to $\omega_1=0.268$, $F'(\omega_1)>0$, and with an upward deflection from this line (e.g., $\Delta\kappa_1>0$, $\Delta\kappa_2=0$), we shall have a positive real increment Δp_1. With this in mind, let us underline this singular line with hatching below to denote that when crossing the singular line downwards, the number of roots of the characteristic equation with a positive real part reduces by 1. Similarly, we conclude that the singular line for ω_2 should be hatched from above, while the singular lines for ω_3, ω_4—in the way as it is shown on Fig. 5, where the picture of D-partitioning is presented.

From the D-partitioning procedure, the region having all the boundaries hatched from within, is only a candidate for the stability region. Therefore, it is necessary to find the real parts of all roots of the characteristic polynomial at least for one point of this region (the couples k_1, and k_2). For this particular case, we do not need to do it since the boundaries of the region are straight lines, corresponding to all the four purely imaginary roots, so under an offset inside the region away from those boundaries, all the roots acquire negative real parts. Therefore, the region of stability is a quadrilateral with its vertexes formed by intersections of the straight lines corresponding to ω_1, ω_2, with those corresponding to ω_3, ω_4.

Coordinates of the vertexes of the quadrilaterals are

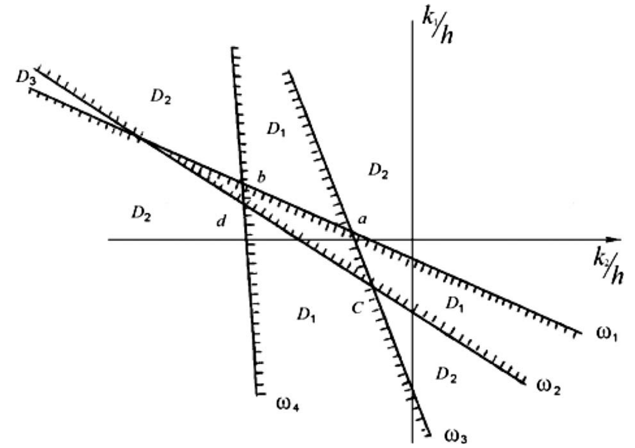

Fig. 5 The picture of D-partitioning of a plane of circulation forces coefficients κ_1, κ_2. Quadrilateral *abcd* is the region of asymptotic stability.

$$\kappa_1 = 0.053h, \quad \kappa_2 = -2.76h; \quad \kappa_1 = 0.199h, \quad \kappa_2 = -3.895h$$

$$\kappa_1 = -0.189h, \quad \kappa_2 = -2.6h; \quad \kappa_1 = 0.079h, \quad \kappa_2 = -3.89h$$

For the sake of stability with a given h, the parameters k_1 and k_2 of the circulation forces should be chosen within this quadrilateral, e.g., $\kappa_1=0$ and $\kappa_2=-3h$.

The area of the quadrilateral increases in accordance with the increasing of h and vanishes at $h\to 0$. From the form of the stability region, it is easy to conclude that the parameters k_1 and k_2, bringing asymptotic stability to the system, are bounded both from above and from below.

It is noteworthy that achieving of asymptotic stability is only possible due to both circulation and dissipation forces. Introduction of either dissipative or circulation forces alone results in instability of gyroscopically stabilized systems.

The possibility of strengthening conservative stability, achieved due to gyroscopic forces up to asymptotic by introducing dissipative and circulation forces, allows to expect that the results of the analysis would be not only theoretically, but also practically interesting. In particular, stabilization of a point charge in electrostatic field due to circulation forces can be implemented in the following way. A system of charges is placed within a cylindrical housing, which is set in rotation. Air drag in the reference frame of the housing is proportional to the body's velocity. After transformation of the resisting force to a fixed reference, in the equations of motion, there will be terms interpretable as circulation forces; therefore, the angular velocity of the housing is the parameter of the circulation forces.

4 The Domain of Attraction of Dynamic Equilibrium of the System

Practical realization of the considered model of a noncontact suspension requires discussion of one more aspect of the problem—maximization of the domain of attraction of the dynamic equilibrium state of the system. For this purpose, it is necessary to solve the problem in the general nonlinear statement (2). Due to the complexity of the nonlinear model, the analysis was performed numerically. The energy conservation law may be used to control precision of the computations. Figure 6 shows the graph of deflection of full energy E from the value of E_0 at the initial time $\Delta E=E-E_0/E_0 100\%$, subject to time t during numerical integration. Deviations of ΔE from zero are insignificant; therefore, the error does not accumulate and almost does not affect the result.

As example, Fig. 7 provides the trajectory of the center of mass on plane xy with the parameters $\chi=10$, $\delta=3$, and $H=6.25$ from

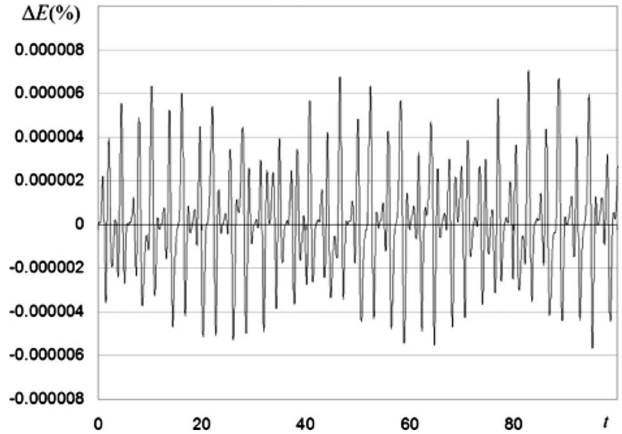

Fig. 6 The deviation of full energy E of the system from initial value E_0 during numerical integration. It is used for the control of calculation accuracy.

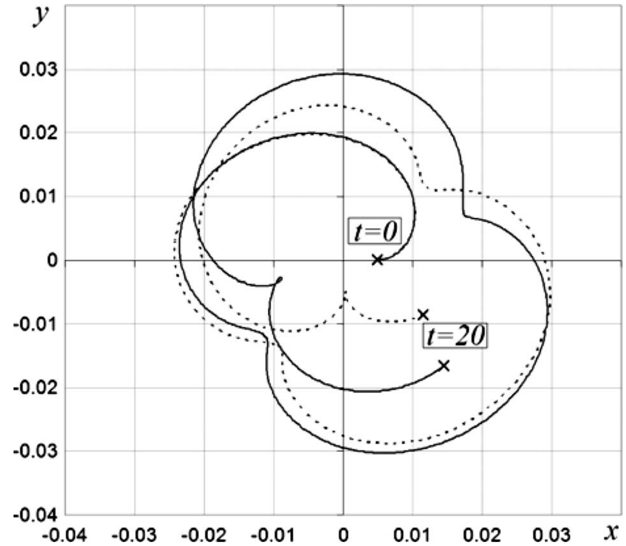

Fig. 7 Trajectory of motion of the center of mass of the body in linear (dotted line) and nonlinear cases with the parameters from the region of stability

the region of conservative stability of the system: the dotted line corresponds to the linear case, the solid line corresponds to the nonlinear.

The dimension of the domain of attraction of the system's equilibrium is 10: three coordinates of the center of mass, two angular coordinates, and their velocities. The results of computation of the domain of attraction in two simplest cases are given below.

Let the initial state be set by three nonzero coordinates, x, y, and z, while their corresponding velocities and angular characteristics are equal to zero. The boundary of the domain of attraction of a stable equilibrium of the system in the plane of $R=\sqrt{x^2+y^2}$, z, computed numerically at $H=6.3$ and $\chi=10$, is shown on Fig. 8(a). The digits mark the curves, corresponding respectively to the values of parameter δ: 3, 3.025, 3.05, 3.075, and 3.1.

The domain of attraction for the case when, in initial state, there are nonzero perturbations in z and in angle a, and the rest of the coordinates and velocities of translational and angular motion are zero, is shown on Fig. 8(b), with the values of the parameters H, χ, and δ, same as in the previous example.

Also, it is possible to study numerically the dependence of the sizes of the domain of attraction on parameters of the systems. The radius $R=\sqrt{x^2+y^2}$ with $z=z_0$ (the distance from 0 to a point of crossing of border of area on Fig. 8(a)) is taken as a measure for the size of the domain of attraction. The changes in size for different parameters δ and χ at $H=6$ are shown with level lines on Fig. 9. The external contours on these figures are the corresponding sections of the region of stability (Figs. 4).

5 Conclusion

The study carried out has allowed to understand on simple model the features of dynamics of a rigid body in the noncontact magnetic suspension of the type magnetic top—Levitron. This work differs from many papers on Levitron dynamics by the exact notation of equations of motion without any simplifications. Also in this work, small nonconservative forces that make stability asymptotic are indicated. The results derived here can form a basis for creation of an electrostatic suspension of this type. Note that levitation can be realized only if positions of the charge and mass center do not coincide.

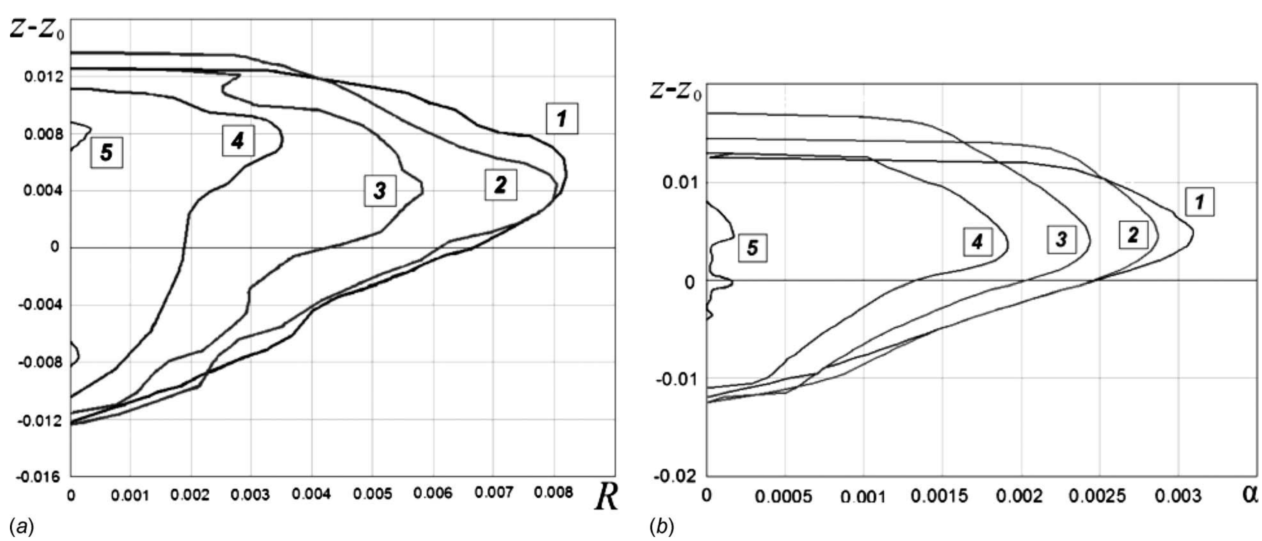

Fig. 8 The domain of attraction of a stable equilibrium (a) in the plane of $R=\sqrt{x^2+y^2}$, z, and (b) in the plane of α, z

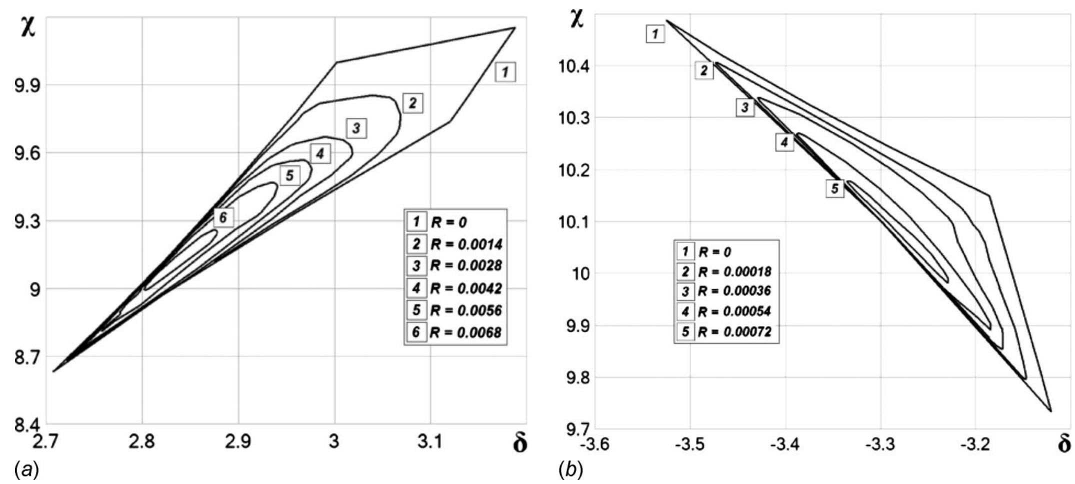

Fig. 9 Dependence of the sizes of domain of attraction on parameters of the system at $H=6$ in the case of (*a*) $\delta>0$ and (*b*) $\delta<0$ correspondingly

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Nomenclature

$\mathbf{r}\ (x,y,z)$ = radius vector of the center of mass of the body
$\mathbf{i},\mathbf{j},\mathbf{k}$ = basis vectors
$\mathbf{r}_1=\mathbf{r}+\mathbf{a}$ = radius vector of the electric charge attached to the body
a = distance between the center of mass and the charge position in the levitating body
α,β = the angles characterizing the position of symmetry axis of the body
C,A = axial and equatorial moments of inertia of the body
m = mass of the body
q = electric charge
$H=C/A\Omega_0$ = parameter of gyroscopic forces
Ω_0 = initial angular speed of rotation of the body
$\delta=a/l_*$ = distance from the charge to the center of mass in the $l_*=\sqrt{A/m}$ scale
$z_0=\delta+q/l_*\sqrt{4\pi\varepsilon_0 mg}$ = vertical coordinate of the center of mass in the state of equilibrium
ε_0 = electric constant
$\mathbf{g}$ = gravitational acceleration
$\chi=z_0\delta$ = coefficient of the overturning moment
κ_1,κ_2 = coefficients of circulation forces
h_1,h_2 = coefficients of dissipative forces

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